CONVECTIVE INSTABILITY IN A HORIZONTAL FLUID LAYER WITH A PERMEABLE BARRIER UNDER CONDITIONS OF ARBITRARY BOUNDARY HEAT CONDUCTION

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We examine the effect on the convective stability of a plane horizontal fluid layer from the resistance of a permeable barrier positioned in the middle of the layer, and from the heat conduction of a solid block surrounding that layer.

Control of the convective equilibrium stability of a fluid is an important technical problem. A method of achieving equilibrium stabilization involves the positioning of permeable barriers within the fluid. Their effect on the stability of a horizontal fluid layer bounded by isothermal plane has been studied in [1, 2]. The estimates carried out in [3], and the experiment described in [4], demonstrated that even rather minor barriers significantly elevate the critical Rayleigh numbers. The mechanism behind this change in stability is closely associated with the effect that the barrier has on the shape of the resulting critical motions. Thus, with low barrier resistances, a convective cell encompasses the entire height of the layer. However, as the barrier resistance increases, local vortices are developed on each side of the barrier, as a consequence of which the horizontal dimension of the cell initially increases, and then, as a two-tiered structure is formed, these perturbations diminish.

Convective stability is also significantly affected by the thermal properties of the fluid layer boundaries. As is well known [5], with a reduction in the thermal conductivity of the boundaries there is a drop in equilibrium stability and an increase in the horizontal dimension of the convective cell. In this particular study we investigate the combined effects of these two factors on the equilibrium stability of the fluid.

Let us examine the equilibrium stability of a layer of a viscous fluid, contained between horizontal planes $z = \pm h$ and separated by a thin permeable barrier $z = 0$. The layer is bounded by solid masses with identical thermal conductivity, in which a temperature gradient directed downward (against the z axis) is maintained. In the absence of convection, a temperature difference of 2Θ arises across the boundaries of the layer.

The problem dealing with the behavior of small normal perturbations can be formulated for the amplitude of the vertical velocity component v(z) and the temperature perturbaton amplitude $\theta(z)$ within the fluid layer. In the absence of a barrier, according to [5], the boundary-value problem for neutral perturbations in dimensionless variables has the following form (the prime denotes differentiation with respect to z):

$$v^{IV} - 2k^2v'' + k^4v = k^2 \,\mathrm{Ra}\,\theta, \quad \theta'' - k^2\theta = -v, \tag{1}$$

$$z = \pm 1: \ v = v' = 0, \ \varkappa\theta' = \mp k\theta. \tag{2}$$

Problem (1), (2) contains the following dimensionless parameters: the Rayleigh number $\mathrm{Ra} = g\beta\Theta h^3/\nu\chi$; the wave number k, defining the periodicity of the perturbations in the horizontal direction; the ratio of the thermal conductivity of the fluid to the thermal conductivity of the block masses is given by κ.

The effect of a thin barrier on the perturbations can be described through substitution of the boundary conditions which exist for $z = 0$. If we regard the barrier as a fine-grained boundary, following [6], we will formulate the averaged boundary conditions prevailing at that boundary.

We will assume that the barrier exhibits no thermal inertia and that temperature and the heat flow at that barrier are continuous:

$$z = 0: \ \theta_- = \theta_+, \ \theta'_- = \theta'_+, \tag{3}$$

where the signs — and + indicate the values of the functions respectively below and above the barrier

We further assume at the permeable barrier continuity of the normal and tangential velocity components and that they are proportional to the sum of the corresponding stresses on either side of the barrier. Using the Navier—Stokes equation and the equation of continuity, we can write the boundary conditions for the amplitude of the vertical velocity component in the form [2]

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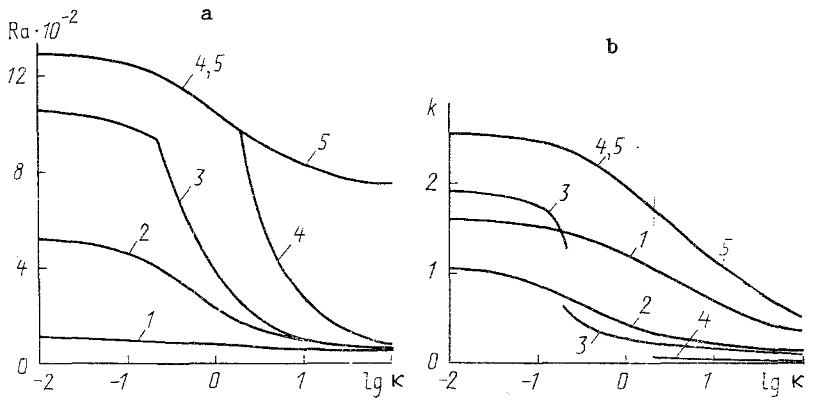

Fig. 1. Critical Rayleigh number $Ra = g\beta\Theta h^3/\nu\chi$ (a) and the wave number in units of h^{-1} (b) as functions of the relative heat conduction of the fluid $\kappa = \kappa_l/\kappa_m$ for various values of normal barrier resistance α_n in units of η/h: 1) $\alpha_n = 0$; 2) 100; 3) 400; 4) 10^4; 5) ∞.

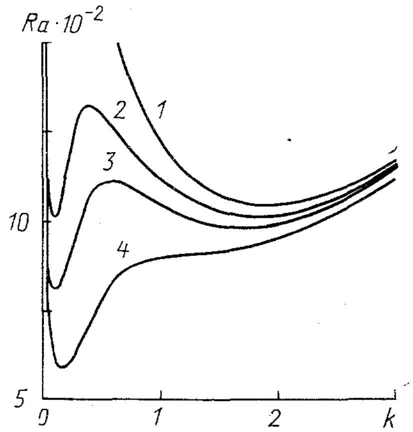

Fig. 2. Neutral curves for the relative heat conduction $\kappa = 1$ of the fluid at various normal barrier resistances α_n in units of η/h: 1) $\alpha_n = 10^4$; 2) 3160; 3) 2000; 4) 1000. $Ra = g\beta\Theta h^3/\nu\chi$; k, h^{-1}.

$$z = 0:\ v_- = v_+,\ v'_- = v'_+,\ v'''_+ - v'''_- = -\alpha_n k^2 v,\ v''_+ - v''_- = \alpha_\tau v'. \tag{4}$$

The coefficients α and α_τ, characterizing the averaged effect of the barrier on the fluid flow, in terms of their sense represent the normal and tangential resistance of the fluid. For the measurement units we have taken the ratio of dynamic viscosity to the half-thickness of the layer.

The uniform boundary-value problem (1)-(4) makes it possible to find the critical Rayleigh numbers at which convection begins, and also the form of the critical motions.

In view of the symmetry of the problem, the perturbations break down into two classes: even [$v(-z) = v(z)$] and odd [$v(-z) = -v(z)$], which we will construct in the region [0, 1]. The boundary conditions in the middle of the layer will be reformulated to account for the even properties of the functions. The conditions of continuity for the even functions are satisfied automatically, while the value of each odd function for $z = +0$ is assumed to be equal to half the discontinuity defined by conditions (3) and (4).

For the even solution we have

$$z = +0:\ v' = 0,\ v''' = -\frac{1}{2}\alpha_n k^2 v,\ \theta' = 0. \tag{5}$$

For the noneven solution from (3) and (4) it follows that

$$z = +0:\ v = 0,\ v'' = \frac{1}{2}\alpha_\tau v',\ \theta = 0. \tag{6}$$

As we can see from boundary conditions (5) and (6), the critical Rayleigh number for the even perturbation depends exclusively on the normal resistance of the barrier, while in the case of noneven perturbation it depends on the tangential resistance. Let us also note that the normal resistance of the grid enters the problem in the form of the parameter $\alpha_n k^2$, so that for long wave even perturbations the resistance of the grid will therefore effectively be reduced.

Equations (1) were integrated numerically by the Runge—Kutta method. Three linearly independent solutions were constructed to satisfy the boundary bonditions for z = 1. The linear combination of these solutions for z = 0 must satisfy boundary conditions (5) or (6). The critical Rayleigh numbers were found from the condition of solvability for the derived algebraic system of equations for weighted coefficients.

Calculations showed that in the entire range of variations in problem parameters the most dangerous are the even perturbations. The critical Rayleigh number and the wave number of the even perturbations, as functions of the relative thermal conductivity of the fluid, are shown in Fig. 1 for various barrier resistances.

In the limit cases of an absolutely permeable ($\alpha_n = 0$) and impermeable ($\alpha_n = \infty$) barrier, according to [5], the increase in the thermal conductivity of the fluid leads to a reduction in the stability of mechanical equilibrium. In these cases, the neutral curves Ra(k) exhibit a single minimum, which as $\kappa \to \infty$ is displaced toward the k = 0 axis.

With low barrier resistances, the most dangerous perturbations are the ones in which circulation encompasses the entire layer. Therefore, the nature of the relationship between the critical parameters and κ remains the same as in the case when $\alpha_n = 0$, although stability is elevated as the barrier resistance increases.

As demonstrated by calculations, for larger barrier resistances ($\alpha_n \geq 400$) the form of the perturbations depends significantly on the wave number. With $k \approx 1$, convective motion exhibits a two-tiered structure, whereas when $k << 1$ the convective vortex occupies both halves of the layer. At a certain value for the thermal conductivity of the fluid, i.e., $\kappa^*(\alpha_n)$, competition is established between these forms of motion. On the neutral curve we observe two minima, and on transition through κ^* the critical wave number undergoes a discontinuous change. Thus, for $\alpha_n = 10^4$ the transition to long wave instability with an increase in the heat conduction of the fluid occurs at $\kappa^* = 1.96$, with the wave number changing from 1.64 to 0.051. Let us note that the wave number of the critical perturbations in the case of long wave instability for large resistances on the part of the permeable barrier turns out to be considerably smaller than in the case in which there is no barrier.

The transition from the shortwave instability to the longwave instability with a change in barrier resistance for the case in which $\kappa = 1$ is illustrated in Fig. 2. For resistances $\alpha_n = 10^4$ we have a shortwave instability with k = 1.9. On this same neutral curve we also have a local minimum Ra = 1758, k = 0.055. With a reduction n the barrier resistance the longwave minimum of the neutral curve drops off rapidly, whereas its shortwave branch shows only slight deformation. When $\alpha_n = 3160$ the instability changes into longwave perturbations.

The sharp drop in stability on transition to longwave perturbations for grids with large concentration can also be seen in Fig. 1. For any α_n on transition through κ^* the critical Rayleigh number rapidly approaches the critical value for the case $\alpha_n = 0$. A special case is the impermeable barrier ($\alpha_n = \infty$) for which transition to the longwave critical perturbations occurs slowly.

Attainment during the experiment of a discontinuous jump in the transition to the longwave instability in the case of high barrier resistance can be eliminated through an insufficient horizontal dimension for the fluid layer, the critical perturbation remaining the same as in the case of an impermeable barrier.

NOTATION

g, free-fall acceleration; β, ν, η, and χ, coefficients of thermal expansion, kinematic viscosity, dynamic viscosity, and thermal diffusivity; h, half the thickness of the layer; Θ, the half-difference of the temperatures at the boundaries of the fluid layer; κ_l and κ_m, the coefficients of thermal conductivity for the fluid and for the solid mass block; $\kappa = \kappa_l/\kappa_m$; k, the wave number for the perturbations, expressed in units of h^{-1}; Ra, the Rayleigh number; α_n and α_τ, normal and tangential resistance of the permeable barriers; z, vertical coordinate, reckoned from the middle of the fluid layers; v and θ, amplitudes of the normal perturbations of the vertical components of velocity and temperature in the fluid layer.

LITERATURE CITED

1. R. V. Birikh and R. N. Rudakov, *Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza*, No. 1, 157-159 (1977).
2. R. V. Birikh and R. N. Rudakov, in: *Convective Flows* [in Russian], Perm' (1981), pp. 12-17.
3. R. V. Birikh and R. N. Rudakov, in: *Convective Flows* [in Russian], Perm' (1989), pp. 81-86.
4. M. Yu. Larkin and M. P. Sorokin, in: *Convective Flows* [in Russian], Perm' (1981), pp. 18-21.
5. G. Z. Gershuni and E. M. Zhukhovitskii, *Convective Instability in an Incompressible Fluid* [in Russian], Moscow (1972).
6. V. A. Marchenko and E. Ya. Khruslov, *Boundary-Value Problems in Areas with Fine-Grained Boundaries* [in Russian], Kiev (1974).